Chapter Summary

Chapter Summary: Motion Along a Straight Line

Kinematic Properties

Kinematics is a description of motion of objects and Kinematic properties are used to describe the motion. The most important properties we will be using are:

- **Position** - The point in space of an object at a specific time, $\vec{x}(t)$
- **Displacement** - The change in position of an object during some interval in time, $\Delta\vec{x} = \vec{x}(t_\mathrm{f}) - \vec{x}(t_\mathrm{i})$ (Equation 3.1)
- **Velocity** - The rate and direction of the change in position with time, at some time, $\vec{v}(t) = \frac{\mathrm{d}}{\mathrm{dt}}\vec{x}(t)$ (Equation 3.8)
- **Average Velocity** - Not as important as the velocity, the displacement of an object divided by the time interval, $\vec{v}_\mathrm{ave}(t) = \frac{\Delta\vec{x}}{\Delta t}$ (Equation 3.6)
- **Acceleration** - The rate and direction of the change in velocity with time, at some time, $\vec{a}(t) = \frac{\mathrm{d}}{\mathrm{dt}}\vec{v}(t)$ (Equation 3.18)
- **Average Acceleration** - The change velocity of an object over some time interval, divided by the time interval, $\vec{a}_\mathrm{ave} = \frac{\Delta\vec{v}}{\Delta t}$ (Equation 3.16)

One-Dimensional Motion

Motion along a straight line is described by a one-dimensional coordinate system. This consists of a defined **Origin**, the point relative to which positions are determined, and a defined positive direction. This direction is defined by a **unit vector** pointing in the positive direction, $\hat{i},\ \hat{j}$, etc. So, for example, the position of an object is a vector from the origin of the coordinate system to the point where the object is at some time, written as

$$\vec{x}(t) = x(t)\hat{i}$$

where $x(t)$ is the "x-component of the position vector". $x(t)$ will be positive if the postion $\vec{x}$ is in the same direction as $\hat{i}$ or in the opposite direction. Similarly, $\vec{v}(t) = v(t)\hat{i}$ and $\vec{a}(t) = a(t)\hat{i}$.

When in one dimension, we can just work with the components, $x(t),\ v(t),\ a(t)$, understanding that the sign of the components give the direction of the vectors.

Constant Velocity and Constant Acceleration Models

For some physical problems, it is possible to approximate the motion as either constant velocity or constant acceleration.

For constant velocity:

Constant Velocity Model (1-D)

If an object is moving with a constant velocity, starting at time t_i at a position $x_i = x(t_i)$, for any $t > t_i$ for which the velocity doesn't change:

$$v = v_{\text{ave}} = \frac{x(t) - x(t_i)}{t - t_i} \tag{3.9}$$

This can be re-written to give the position of the object at a later time:

$$\text{1-D Motion: } x(t) = v \cdot (t - t_i) + x_i \tag{3.10}$$

For constant acceleration:

Constant Acceleration Kinematic Equations in One Dimension

1. The position of an object as a function of time and the final position of the object for a time interval $\Delta t = t_{\mathrm{f}} - t_{\mathrm{i}}$:

$$\begin{aligned} x(t) &= \tfrac{1}{2}a \cdot (t_{\mathrm{f}} - t_{\mathrm{i}})^2 + v_{\mathrm{i}} \cdot (t_{\mathrm{f}} - t_{\mathrm{i}}) + x_{\mathrm{i}} \\ x_{\mathrm{f}} &= \tfrac{1}{2}a \cdot \Delta t^2 + v_{\mathrm{i}} \cdot \Delta t + x_{\mathrm{i}} \end{aligned} \tag{3.31}$$

2. The velocity of the object as a function of time and the final velocity for a time interval $\Delta t = t_{\mathrm{f}} - t_{\mathrm{i}}$:

$$\begin{aligned} v(t) &= a \cdot (t_{\mathrm{f}} - t_{\mathrm{i}}) + v_{\mathrm{i}} \\ v_{\mathrm{f}} &= a \cdot \Delta t + v_{\mathrm{i}} \end{aligned} \tag{3.32}$$

3. The position of the object as a function of time and the final position for a time interval $\Delta t = t_{\mathrm{f}} - t_{\mathrm{i}}$ in terms of the average velocity, without using the acceleration:

$$\begin{aligned} x(t) &= \tfrac{1}{2} \cdot (v(t) + v_{\mathrm{i}}) + x_{\mathrm{i}} \\ x_{\mathrm{f}} &= \tfrac{1}{2} \cdot (v_{\mathrm{f}} + v_{\mathrm{i}}) + x_{\mathrm{i}} \end{aligned} \tag{3.33}$$

4. The relation between positions and velocities of the object, without explicitly including the time interval: